Chapter 8: Polynomials and Factoring

Dear Family,

Your child will learn a variety of methods for factoring polynomials.

The numbers or expressions that are multiplied together to create a product are called *factors* of that product. Most whole numbers can be factored several different ways. Here are four ways to factor 18.

Factorizations of 18: $1 \cdot 18$ $\quad 2 \cdot 9$ $\quad 3 \cdot 6$ $\quad 2 \cdot 3 \cdot 3$

However, any whole number has only one **prime factorization** made of all prime numbers. (A prime number has exactly two factors, itself and 1.) For 18, the prime factorization is $2 \cdot 3 \cdot 3$.

The **greatest common factor** (GCF) of two or more monomials is the greatest factor common to all of the expressions. One method of finding the GCF is to align the common prime factors and find the product of them. When the expressions include variables, write any powers as products.

Find the GCF of $24x^3$ and $60x^2$.

Prime factorization of $24x^3$: $\boxed{2} \cdot \boxed{2} \cdot 2 \cdot \boxed{3} \cdot \boxed{x} \cdot \boxed{x} \cdot x$

Prime factorization of $60x^2$: $\boxed{2} \cdot \boxed{2} \cdot \boxed{3} \cdot 5 \cdot \boxed{x} \cdot \boxed{x}$

Common factors: $2 \cdot 2 \cdot 3 \cdot x \cdot x = 12x^2$

The GCF of $24x^3$ and $60x^2$ is $12x^2$.

The Distributive Property states that *a* (*b* 1 *c*) 5 *ab* 1 *ac*. Because *a* becomes the GCF of both terms of the polynomial, you can use the distributive property in "reverse" to *factor* it out again: *ab* 1 *ac* 5 *a (b* 1 *c)*.

Factor $24x^3 - 60x^2$.

$$24x^3 - 60x^2 = \underbrace{2x(12x^2) - 5(12x^2)}_{\text{Write the terms with the GCF as a factor.}} = \underbrace{12x^2(2x - 5)}_{\text{Factor out the GCF.}}$$

When a polynomial contains four terms, you may be able to *factor by grouping*. This method is shown below.

Factor $6b^3 + 8b^2 + 9b + 12$.

$(6b^3 + 8b^2) + (9b + 12)$ *Group terms.*

$2b^2(3b + 4) + 3(3b + 4)$ *Factor the GCF of each group.*

$(3b + 4)(2b^2 + 3)$ *Factor out the common binomial.*

When you use the FOIL method to multiply two binomials, you may notice patterns in the coefficients of the resulting trinomial. For example:

$$(x + 5)(x - 8) = x^2 - 8x + 5x - 40 = x^2 - 3x - 40$$

−3 is the sum of 5 and −8.

−40 is the product of 5 and −8.

So, to factor a trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b.

Factor $x^2 + 12x + 20$.

Factor of 20	Sum	
1 and 20	21	▯
2 and 10	12	✓
4 and 5	9	▯

2 and 10 are the only factors that satisfy the pattern.

$$x^2 + 12x + 20 = (x + 2)(x + 10)$$

Now consider what happens when you use the FOIL method on two binomials that have leading coefficients:

8 is the product of 2 and 4.

3 is the product of 3 and 1.

$$(2x + 3)(4x + 1) = 8x^2 + 2x + 12x + 3 = 8x^2 + 14x + 3$$

14 is the sum of $(2 \cdot 1) + (3 \cdot 4)$.

So, to factor a trinomial of the form $ax^2 + bx + c$, find factors of a and c, and then check that the products of the outer and inner terms sum to b.

Factor $5x^2 - 14x + 8$.

Factors of 5	Factors of 8	Outer + Inner	
1 and 5	−1 and −8	$1(-8) + 5(-1) = -13$	▯
1 and 5	−8 and −1	$1(-1) + 5(-8) = -41$	▯
1 and 5	−2 and −4	$1(-4) + 5(-2) = -14$	✓
1 and 5	−4 and −2	$1(-2) + 5(-4) = -22$	▯

Notice that c is positive, so it must be the product of two positive numbers or two negative numbers. But, because b is negative, you only need to try the negative possibilities.

$$5x^2 - 14x + 8 = (x - 2)(5x - 4)$$

The special products introduced in chapter 7 (perfect squares and differences of two squares) lead to two special factoring situations.

Perfect-Square Trinomials	$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
	$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$
Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$

The chapter concludes with a lesson on choosing and combining factoring methods to make sure a polynomial is fully factored.

Sincerely,

Ms. Talath Ansari