Chapter 9: Quadratic Functions and Equations

Dear Family,

Your child will graph quadratic functions, explore the characteristics of quadratic functions, and solve quadratic equations. A **quadratic function** is any function that can be written in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. You can identify a quadratic function when its equation fits the standard form or when a table of ordered pairs has constant *second differences*.

$$y + 2x = 4x^2$$
$$-2x \qquad -2x$$
$$y = 4x^2 - 2x$$

This function is quadratic because it can be written in standard form where $a = 4$, $b = -2$, and $c = 0$.

x	y
−1	6
0	0
1	2
2	12

first differences: -6, $+2$, $+10$

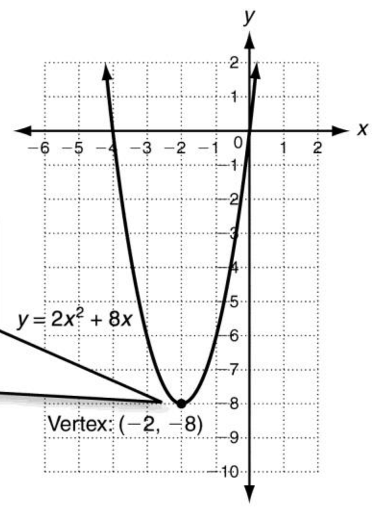
second differences: $+8$, $+8$

This function is quadratic because a constant change in x corresponds to constant second differences of y.

The graph of a quadratic function is a U-shaped curve called a **parabola**. The highest or lowest point on a parabola is called the **vertex**. When the leading coefficient, a, is positive, the y-value of the vertex is the **minimum** value of the function. When a is negative, it is a **maximum**.

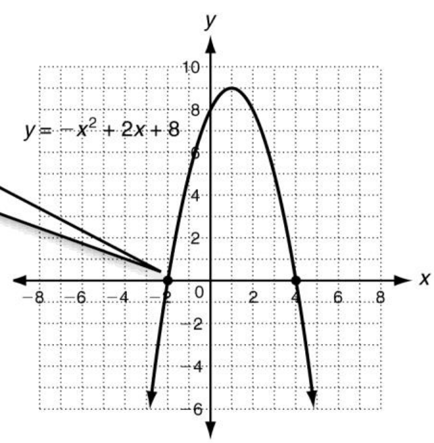
For $y = 2x^2 + 8x$, the value of a is $+2$, so the parabola opens upward.

This makes the vertex the lowest point on the curve, and the minimum value of the function is -8.



A **zero of a function** is an x-value that makes the function equal to 0. A zero of a function is the same as an x-intercept of its graph. A quadratic function may have two, one, or no zeros.

From the graph, the function $y = -x^2 + 2x + 8$ has two zeros, $x = -2$ and $x = 4$.



You may notice that each parabola has a vertical **axis of symmetry** that divides it in half. You may also notice that the axis of symmetry passes through the vertex. If you graph the axis of symmetry, the vertex, and a few points on one side of a parabola, you can then use the parabola's symmetry to graph its other half.

Chapter 8 explores transformations of the parent function $f(x) = x^2$.

Transformed Function		Relationship to the Parent Function $f(x) = x^2$
$f(x) = ax^2$	$\lvert a \rvert > 1$	the parabola is narrower
	$\lvert a \rvert < 1$	the parabola is wider
$f(x) = x^2 + c$	$c > 0$	the parabola is translated *c* units up
	$c < 0$	the parabola is translated *c* units down

Related to quadratic functions are **quadratic equations**: $ax^2 + bx + c = 0$. Because *y* is replaced by 0, the solutions of a quadratic equation are the same as the zeros of a quadratic function. So, one method of solving a quadratic equation is to graph the related function and find its *x*- intercepts. You can solve some quadratic equations by factoring.

Solve $x^2 - 3x - 18 = 0$.

$$(x + 3)(x - 6) = 0$$
$$x + 3 = 0 \text{ or } x - 6 = 0$$
$$x = -3 \text{ or } x = 6$$

Zero Product Property
If the product of two quantities is zero, then at least one of the quantities is zero.

You can solve other quadratic equations by using a square root. The square-root method is most convenient when one side of the equation is a perfect square, like x^2 or $(x + 3)^2$. **Completing the square** is a method of solving a quadratic equation by creating a perfect square.

If you complete the square for the general equation $ax^2 + bx + c = 0$, you get the **Quadratic Formula:**

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sincerely,

Ms. Talath Ansari